



Towards a full model for ocular biomechanics, fluid dynamics, and hemodynamics

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Abstract

This contribution presents an ongoing work to implement a patient-specific mathematical virtual simulator for the eye. The aim is to create a multiscale and multiphysics model for the description of ocular biomechanics, fluid dynamics, and hemodynamics. This instrument may serve to illustrate and estimate some clinically relevant parameters, as well as predict their spatial and temporal evolution adopting forward-looking numerical techniques.

Keywords: hybridizable discontinuous Galerkin method, multiphysics, multiscale, ocular virtual simulator

1. Introduction

Our purpose is to develop a patient-specific Mathematical Virtual Simulator (MVS) that can describe and quantify the interaction between biomechanics, fluid dynamics, and hemodynamics in the eye. The aim is to couple all the various physical aspects in the same model in order to have a full overview of the ocular system. To reach this ambitious goal, we have identified four main modeling steps:

1. blood circulation in the eye from a systemic viewpoint;

2. biomechanics and tissue perfusion in the optic nerve head and the lamina cribrosa;¹
3. biomechanics for the sclera, retina, choroid, and cornea; and
4. fluid dynamics of the vitreous and aqueous humor.

Each step is naturally coupled with the others, giving rise to challenging problems from both the modeling and computational perspectives. In this abstract, we focus our attention on the second and third steps.

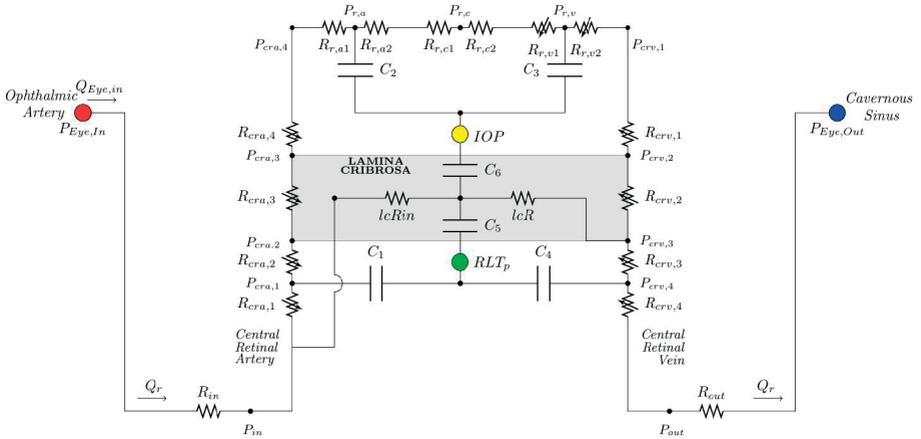


Fig. 1. Circuit-based model retinal and retrobulbar flow.

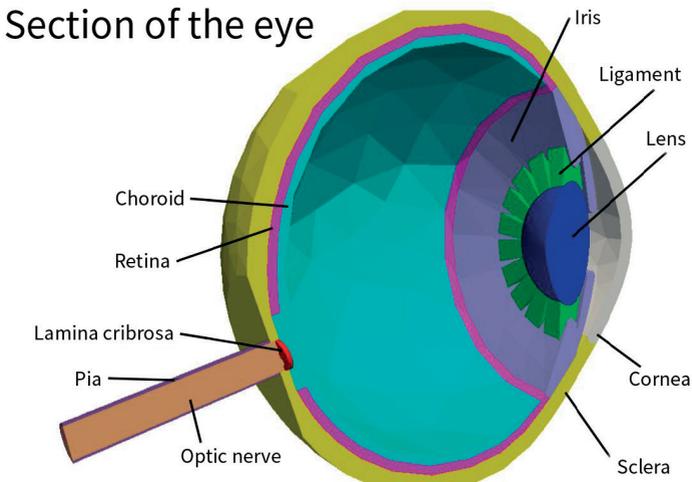


Fig. 2. The 3-D geometry generated with Salome.

2. Model and methods

The MVS has a multiscale architecture that aims at preserving the natural systemic features of blood circulation, while providing detailed views on sites of particular interest from the clinical viewpoint, such as the lamina cribrosa. Thus, MVS combines:

1. a circuit-based model for blood flow in the retinal vasculature, central retinal artery (CRA), and central retinal vein (CRV);
2. a 3-D porous media model for the perfusion of the lamina cribrosa; and
3. a 3-D isotropic elastic model for the biomechanics in the lamina cribrosa, retina, choroid, sclera, and cornea.

Figure 1 illustrates the circuit-based model in which we are substituting the lumped-parameters description of the lamina with a poroelastic spatial distribution model of the lamina. The elastic system of the lamina is also influenced by the biomechanics of the sclera, retina, choroid, and cornea. The novelty of the methods in the current work comes from two complementary perspectives. On the one hand, from a clinical perspective, the model inputs (*e.g.*, blood pressure, intraocular pressure, and ocular geometry) are easily accessible with standard instruments and can therefore be tailored to patient-specific conditions. In particular, starting from an initial computer-aided design (CAD) geometrical model, the structure of the geometry is further elaborated using Salome (OPEN CASCADE SAS, Guyancourt, France)¹ with the help of a Python (Python Software Foundation, The Netherlands) script that incorporates some parameterized values (*e.g.*, thickness of the lamina cribrosa) into the final meshed geometry (Fig. 2). On the other hand, from a numerical viewpoint, the problem involves a non-trivial coupling between biomechanics and hemodynamics in the tissue of the lamina cribrosa, which calls for:

1. high accuracy in the approximation for both primal variables (*i.e.*, displacement and pressure) and dual variables (*i.e.*, stress and perfusion velocity); and
2. integral boundary conditions to account for the coupling between 0-D and 3-D model components.

2.1. Coupling between biomechanics and hemodynamics

The two-way coupling between biomechanics and hemodynamics contributing to tissue perfusion in the lamina cribrosa is taken into account by means of the following poroelastic model:

$$\nabla \cdot \underline{\underline{\sigma}} = f_{el} \tag{1}$$

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_{el} - p \underline{\underline{I}} \tag{2}$$

$$\underline{\underline{\sigma}}_{el} = \mu (\nabla u + \nabla^T u) + \lambda (\nabla \cdot u) \underline{\underline{I}} \quad (3)$$

$$\nabla \cdot \underline{\underline{\sigma}} + \nabla \cdot v = f_{fl} \quad (4)$$

$$v + \kappa \nabla p = 0 \quad (5)$$

where $\underline{\underline{\sigma}}$ is the total stress tensor, μ is the solid displacement, p is the fluid pressure, v is the discharge velocity, λ and μ are the elastic parameters, κ is the permeability, and f_{el} and f_{fl} are volumetric sources of linear momentum and fluid mass, respectively. The poroelastic system, illustrated in Equations (1-5), has been already studied and presented in Causin *et al.*² and Prada.³ Here, we highlight the importance of the two terms in red, since they are responsible for the biomechanical/hemodynamical coupling. Indeed, the presence of capillaries within the lamina collagen beams may influence the biomechanical behavior of the tissue, as expressed in Equation (2). In turn, structural deformations may induce local changes in blood flow, as expressed in Equation (4).

2.2. Hybridizable discontinuous Galerkin method and integral boundary condition

To numerically solve the coupled system of Equations (1-5), we implemented a hybridizable discontinuous Galerkin (HDG) method⁴ in the multiphysics open-source platform Feel++ (Cemosis; Strasbourg, France).⁵ The HDG method has several attractive features:

1. it provides optimal approximation of both primal (pressure, displacement) and dual (velocity, stress) variables;
2. it requires less globally coupled degrees of freedom than DG methods of comparable accuracy using static condensation; and
3. it allows local element-by-element post-processing to obtain new approximations with enhanced accuracy and conservation properties.

Furthermore, to handle the multiscale nature of this problem, we coupled the 3-D system of partial differential Equations (1-5) with a 0-D circuit model implemented in OpenModelica (Open Source Modelica Consortium; Linköping, Sweden). The complex coupling between the HDG system and the circuit has been achieved using integral boundary condition (IBC) and a time-splitting energy-based scheme in the spirit of a recent method presented in Carichino *et al.*⁶ The combination of IBC with the high accuracy in dual variables of the HDG method revealed to be determinant for the proper resolution of the coupled multiscale problem. A detailed explanation of this procedure can be found in Sala *et al.*⁷

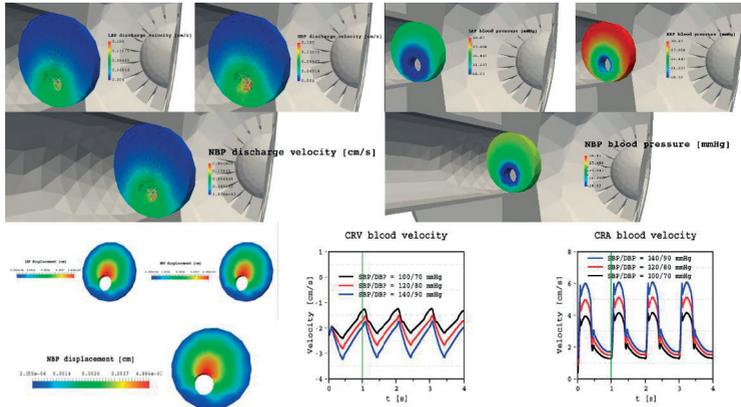


Fig. 3. Lamina cribrosa and CRA/CRV results simulated via MVS.

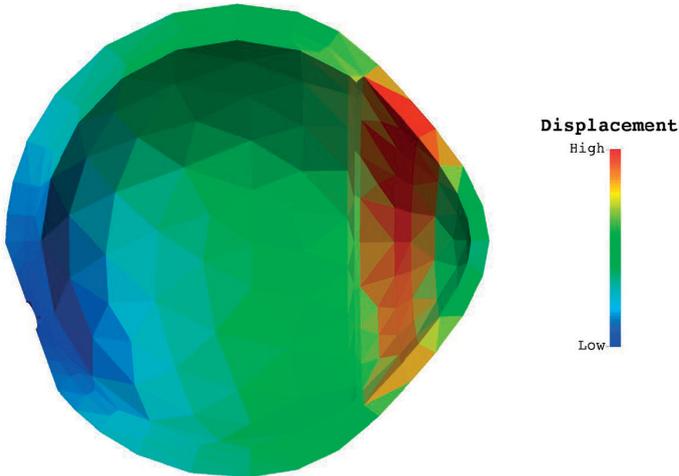


Fig. 4. Qualitative displacement on retina, choroid, sclera, and cornea simulated via MVS.

3. Results

Simulated distributions of velocities and pressure within the lamina, and of blood velocity within the CRA and CRV obtained via the MVS (see Fig. 3) exhibit a satisfactory agreement with reported experimental data,⁸⁻¹⁰ with a special focus on the blood pressure computed on the lateral boundary of the lamina, where we have applied the integral interface condition between the 0-D and the 3-D models.

Moreover, some outcomes (Fig. 3, two bottom right figures) of MVS can be directly compared to those obtained via direct imaging modalities.¹¹

In Figure 4, we show an overview of the qualitative behavior of the displacement for the lamina, retina, choroid, sclera, and cornea.

4. Conclusion

In conclusion, MVS combines innovative and complex methods such as integral boundary condition for HDG in order to be able to address the increasing demand of information on parts of the eye that are not easily accessible with standard research methods, such as those regarding the perfusion of the lamina cribrosa in the optic nerve head. Furthermore, the proposed model develops an interesting multiphysics and multiscale approach to connect different characters such as biomechanics and hemodynamics.

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